**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans : 0.2676

Probability that the service manager cannot meet his commitment =p(X50)=1-pr(X<=50)

Std normal variable Z=(X-mue)/sigma= (X-45)/8

p(X=50)=1-pr(X<=50)

pr(Z<=(50-45)/8)

pr(Z<=0.625)

from the Z table

= 0.73237= 73.23%

probability that the service manager cannot meet his commitment :100-73.23= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans : probability of emp>44= pr(X>44)=1-Pr(X<=44)

Z=(X-mue)/sigma

As per the given data

Pr(<=(44-38/6)=Pr(Z<=1)= 0.84134=84.134%

100-84.134= 15.866

probability of emp>38= pr(X>=38)=1-Pr(X>=38)

Pr(<=(38-38/6)=Pr(z>=0)= 0.5

Probability of emp btw 38&44= Pr(X<=44)-Pr(X>=38)

= 0.84134-0.5= 0.34134 = 34.134%

More employees at the processing center are older than 44 than between 38 and 44. Will be true

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : prob of emp less than 30=pr(X<30)

Z=(X-mue)/sigma=(30-38)/6

Pr(X<30)=pr(z<(30-38/6)= pr(z-1.3333)=.09176=9.17%

Number of emp with prob 0.0917 being under 30 = 400\*0.0917= 36.38=36 which is true

A training program for employees under the age of 30 at the center would be expected to attract about 36 employees will be true

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Prob btw a nd b is 0.99

Prob outside a& b is 1-0.99=0.01

Prob towards left of a = 0.01/2=0.005

Prob towards right of b=0.01/2=0.005

For prob 0.005 the Z value is -2.57

Hence, Z=(X-mue)/sigma

Z=-2.57

Mue=100

Sigma=20

X =?

Hence to find X, we can use Z\*sigma+mue=X

-(-2.57)\*20+100=151.4

(-2.57)\*20+100=48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

* Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

> qnorm(0.025,45\*3,3)

[1] 129.1201

> qnorm(0.975,45\*5,3)

[1] 230.8799

> qnorm(0.025,45\*7,3)

[1] 309.1201

> qnorm(0.975,45\*7,3)

[1] 320.8799

So the Rupee range with 95% probability for the annual profit of the company is given by,

=Profit1 + Profit2

= [219.12, 230.87] + [309.12, 320.87]

= [528.24, 551.74]

So Profit of the company in Rupees(in **Million**) is between range **[528.24, 551.74]**

* Specify the 5th percentile of profit (in Rupees) for the company

> qnorm(0.05,45\*7,3)

[1] 310.0654

> qnorm(0.05,45\*5,3)

[1] 220.0654

5th percentile of profit = 310.0654+ 220.0654 = **530.1308 Million Rs**

* Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 2 with distribution N(7, 42)